Example Suppose Austin throws a baseball with initial velocity 75 miles/hr, at an angle of 30° from the horizontal, starting at a height of 5.0 ft.

1. When does the ball reach its maximum height?
2. What is the maximum height?
3. When does the ball hit the ground?
4. How far did the ball go?

Horizontal motion (without external forces) is constant velocity, zero acceleration.

Let's think about this:

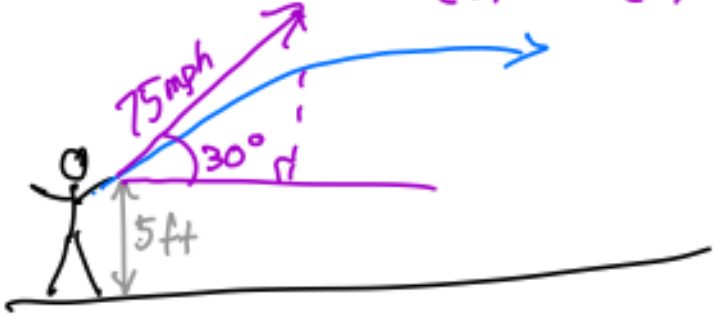
Vertical position $s(t) = ??$

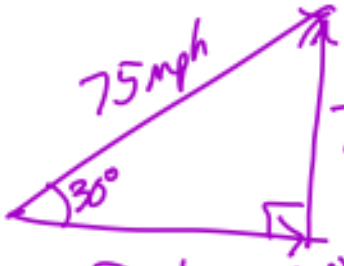
$$s''(t) = a(t) = -32 \text{ ft/sec}^2$$

$$\Rightarrow s'(t) = v(t) = -32t + C$$

(C: constant)

the only fcn so that $v'(t) = -32$.





$75\text{mph} \cdot \sin(30°) = 37.5 \frac{\text{miles}}{\text{hour}} = 37.5 \frac{5280\text{ft}}{3600\text{s}}$

$= 55 \text{ ft/s}$

$75 \text{ mph} \cos(30°) = 75 \text{ mph} \cdot \frac{\sqrt{3}}{2} = \frac{75\sqrt{3}}{2} \frac{\text{miles}}{\text{hour}} = \left(\frac{75\sqrt{3}}{2}\right) \frac{5280\text{ft}}{3600\text{s}}$

$= 95.3 \text{ ft/s}$

→ When $t = 0$

$v(t) = 55 \text{ ft/s}$

$v(t) = -32t + C = 55$

$\Rightarrow C = 55 \text{ ft/s}$

vertical velocity.

$$\boxed{v(t) = -32t + 55}$$

(a) at maximum height $v(t) = 0$

$-32t + 55 = 0$

$55 = 32t \quad t = \frac{55}{32}\text{s}$

$$\Rightarrow \boxed{t = 1.71\text{s}}$$

(b) What is $s(1.71\text{s})$? maximum height.

(c) $s(t) = 0$ what is t?

→ Need an equation for $s(t)$ = vertical position.

We know $s'(t) = v(t) = -32t + 55$ + 0

what could $s(t)$ be? ↑ derivation ↑

$s(t) = -16t^2 + 55t + C_2$

How can we figure out C_2?

Starting height $= s(0) = 5.0\text{ ft} = -16(0)^2 + 55(0) + C_2$

($t = 0$)

$C_2 = 5.0\text{ ft}$!

$\Rightarrow s(t) = -16t^2 + 55t + 5$

(b) Max height $= s(1.71) = -16(1.71)^2 + 55(1.71) + 5.0$

$= 52.3\text{ ft}$

(-16: ft/s²; t: s; 55: ft/s; t: s; 5.0: ft)

(c) What is t when $s(t) = 0$?

$-16t^2 + 55t + 5 = 0$

$\Rightarrow t = \dfrac{-55 \pm \sqrt{(55)^2 - 4\cdot(-16)(5)}}{2\cdot(-16)}$

$\boxed{t = 3.53\text{ s}}$

(d) How far did the ball go?

Horizontal velocity $= 95.3\text{ ft/s}$

Horizontal position when it hits the ground

$= (95.3\text{ ft/s})(3.53\text{ s}) = \boxed{336.4\text{ ft}}$